

Final Exam Practice Problems:

Answers

1. The top three sales agents in one company are comparing their car sales from last year. Gina sold 160 less than twice the cars that Hannah sold, and John sold 25 fewer cars than Hannah. If the three of them sold 515 cars altogether, then how many did each sell?

a) Identify and label the variables and the constants in this problem, including units.

G = # of cars Gina sold
H = # of cars Hannah sold
J = # of cars John sold

b) Write an equation or equations that will help you solve the problem algebraically.

$$G + H + J = 515 \quad J = H - 25$$
$$G = 2H - 160$$

c) Solve the problem algebraically (using your variables and the equation(s)).

$$(2H - 160) + H + (H - 25) = 515$$

$$4H - 185 = 515$$

$$4H = 700$$

$$H = 175 \text{ cars}$$

$$G = 2(175) - 160$$
$$= 350 - 160$$
$$G = 190 \text{ cars}$$

$$J = 175 - 25 = 150$$

$$J = 150$$

2. Is the relation shown in each table a function? Explain why or why not.

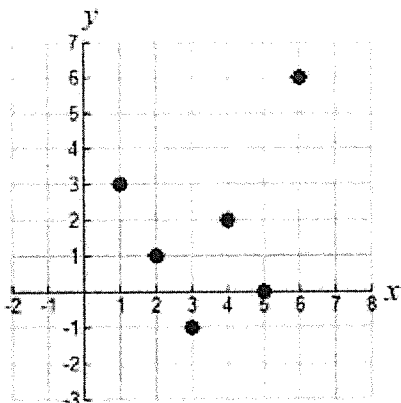
age	# visits to Disneyland
2	0
5	1
9	3
12	2
9	4
11	2

No, the input of 9 has two different outputs.

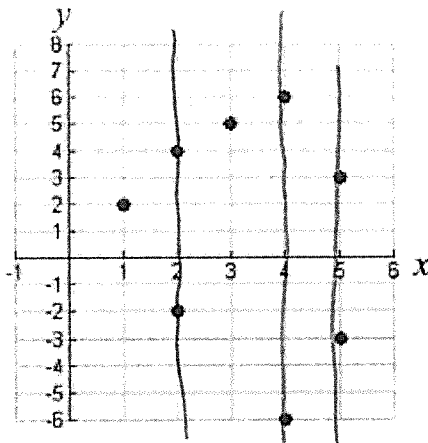
# folds	# layers thick
0	1
1	2
2	4
3	8
4	16
5	32

Yes, each input has exactly one output.

3. Explain why each graph does or does not represent a function.



Yes, each input has exactly one output.



No, the inputs 2, 4 and 5 each have 2 outputs. Does not pass the vertical line test.

4. For each linear situation below:

- Identify and label the variables and the constants including units.
- Identify the constant rate of change and the initial value.
- Write an equation for the linear function that relates the two variables.

I. Linda bought a seedling plant that is two feet ^{24 inches} tall and will grow about $\frac{1}{4}$ inch per week.

a) x = number of weeks

$H(x)$ = Height in inches

Starts 24 in tall, Grows $\frac{1}{4}$ in per week

b) Rate of change = $\frac{1}{4}$ inch per week

c) $H(x) = \frac{1}{4}x + 24$

II. A car rental company charges \$25 to rent and 2 cents for every mile driven.

a) $T(m)$ = Total cost in dollars
 m = miles driven \$25 initial value

c) $T(m) = .02m + 25$

b) Rate of change = \$.02 per mile

III. The pool started with 30,000 gallons of water in it and then drained 12 gallons every 3 minutes.

a) $G(m)$ = Gallons in pool
 m = time pool is draining in minutes
 30,000 initial value

c) $G(m) = -4m + 30,000$

b) Rate of change = $\frac{-12 \text{ gallons}}{3 \text{ mins}} = -4 \text{ gallons per minute}$

5. Some college students who plan on becoming math teachers decide to set up a tutoring service for high school math students. One student was charged \$25 for 3 hours of tutoring. Another student was charged \$55 for 7 hours of tutoring. The relationship between the cost and time is linear.

- a) Identify the independent and dependent variables including their units.

h = hours of tutoring
 $C(h)$ = Amt Charged in dollars

- b) Create a table of values for the situation. Write an equation to model the situation

h	$C(h)$
3	25
7	55

+4 < > +30

Rate of change $\frac{30}{4} = \$7.5$ per hour

$$C(h) = 7.5h + 2.5$$

$$\begin{aligned} 25 &= 7.5(3) + b \\ 25 &= 22.5 + b \\ 2.5 &= b \end{aligned}$$

- c) What is the meaning of the slope in this situation?

They charge \$7.50 per hour

- d) What is the cost-intercept? What does the cost-intercept represent?

\$2.50 which is the initial charge for any tutoring session before the session starts

- e) Show how to use your function equation to find the cost of a 5-hour tutoring session.

$$C(h) = 7.5h + 2.5$$

$$C(5) = 7.5(5) + 2.5$$

$$C(5) = 40$$

$$\boxed{\$40}$$

6. The cost of a rental house for a vacation is the cleaning fee plus the cost for each night you stay. The cost in dollars when you stay x nights is given by the function:

$$C(x) = 150 + 200x$$

- a) What is the cleaning fee?

\$150

- b) Explain the meaning of the coefficient 200 in the equation using units.

\$200 per night

- c) If the total cost is \$1750, then how many nights did you stay?

$$1750 = 150 + 200x$$

$$\frac{1600}{200} = \frac{200x}{200}$$

$$x = 8$$

8 nights

- d) If you can go on vacation for at most 14 days, then what is the domain and range of this function?

$$C(0) = 150$$

$$C(14) = 150 + 200(14)$$

$$150 + 2800$$

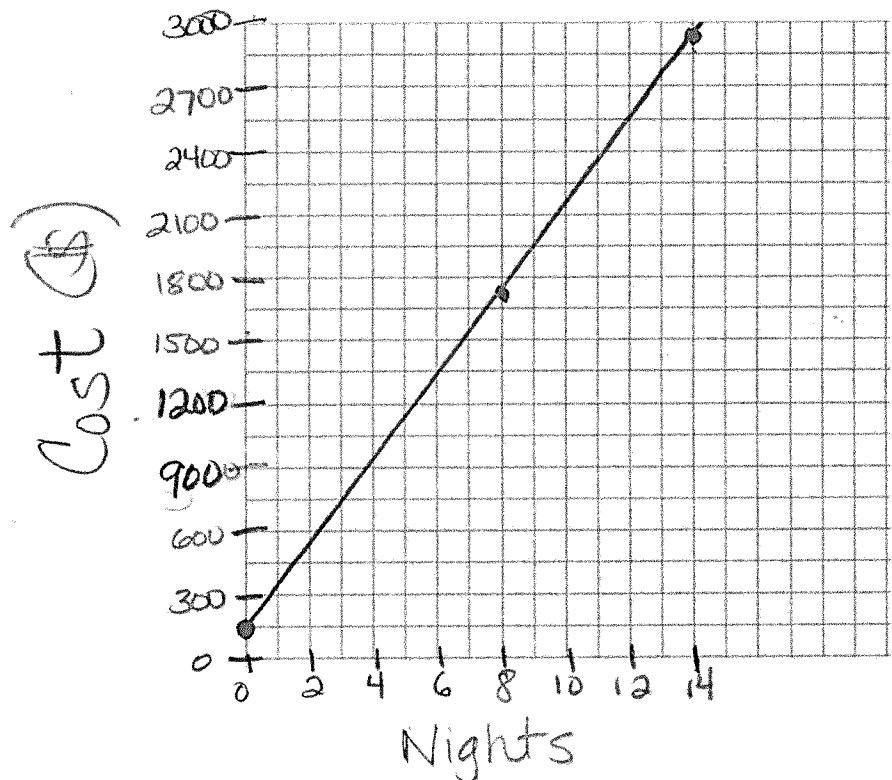
$$\$2950$$

$$D [0, 14]$$

$$R [150 - 2950]$$

- e) Draw a graph of the function with the domain and range from (d) by plotting at least 3 points. Label axes with variables and units and choose appropriate scales.

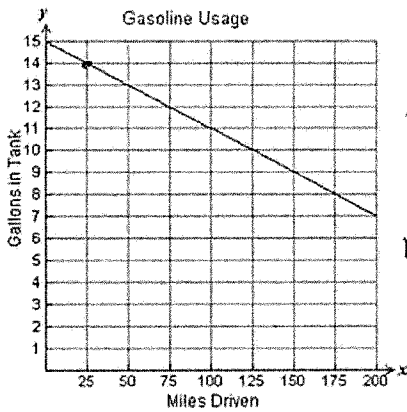
$(0, 150)$
 $(8, 1750)$
 $(14, 2950)$



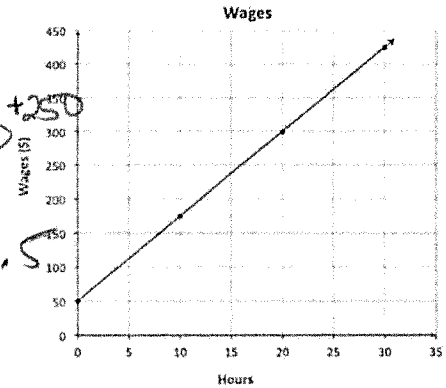
7. Answer the following questions for each relation below:

- Identify and label the dependent and independent variables in each relation below including units.
- Find and interpret the meaning of the slope including units.
- Write an equation that expresses the relationship between the variables.
- Explain the meaning of the y intercept in each graph.

a) The graph shows the number of gallons of gas left in a tank after driving a certain number of miles.



b) The graph shows the weekly wages an employee can earn at Company X.



$$\begin{array}{r} x \ y \\ 25 \overline{) 0 \ 15} \\ \underline{25} \ 14 \end{array} > -1$$

Rate of Change
 $-\frac{1}{25}$ gallons per mile

$$\begin{array}{r} x \ y \\ 20 \overline{) 0 \ 50} \\ \underline{20} \ 300 \end{array} > +250$$

$$m = \frac{250}{20} = 12.5$$

- IV = miles driven
- DV = Gallons of gas in the tank
- Slope = Use .04 gallons per mile
- $y = -0.4x + 15$
- $y = -\frac{1}{25}x + 15$
- y-intercept starts with 15 gallons of gas initially

- IV = Hours worked
- DV = Wages in dollars
- Slope = \$12.5 per hour
- $y = 12.5x + 50$
- y-intercept - Base Pay of \$50 before starting work.

8. Julia just made a purchase on her credit card and can't pay it off. The function that models the amount she owes in dollars, t years after her purchase is:

$$A(t) = 750(1.18)^t$$

a) How much did Julia charge on her credit card?

$$\$750$$

b) How much does Julia owe after one and a half years?

$$A(1.5) = 750(1.18)^{1.5} = \$961.36$$

c) What is the interest rate on this credit card?

$$18\%$$

d) Show how to use logarithms to find out how long it will take for her to owe twice as much as she originally charged. Round to the nearest year.

$$1500 = 750(1.18)^t$$

$$2 = (1.18)^t \rightarrow \frac{\log 2}{\log 1.18} = 4.18$$

4 years

9. A doctor administers 600 mg of medicine to a patient. The level of the drug in the bloodstream decreases by 6% every hour. .94

a) Identify the independent and dependent variables including their units.

$h = IV = \text{hours since administered}$

$m(h) = DV = \text{medicine left (mg)}$

b) Is this a linear, exponential or quadratic function? Write an equation for the function that relates the two variables.

$$m(h) = 600(.94)^h$$

c) How much medicine is in the bloodstream after 4 hours?

$$m(4) = 600(.94)^4 = 468.45$$

d) If the patient needs at least 150 mg in her blood, after how many hours should she get another dose?

$$\frac{150}{600} = \frac{600(.94)^h}{600}$$

$$.25 = (.94)^h$$

$$\frac{\log .25}{\log .94} = 22.4$$

$\approx 22 \text{ hours}$

10. Given the function: $f(x) = x^2 + 12x + 48$

Find the following important points and put them in a table. Show and label your work clearly.

a) Find $f(10)$. Write your answer as an ordered pair.

$$(10, 268)$$

$$\begin{aligned} f(10) &= (10)^2 + 12(10) + 48 \\ &= 100 + 120 + 48 \\ &= 268 \end{aligned}$$

b) Find the vertex.

$$x = \frac{-b}{2a} = \frac{-12}{2} = -6$$

$$f(-6) = (-6)^2 + 12(-6) + 48 = 36 - 72 + 48 = 12$$

$$(-6, 12)$$

c) Find the y-intercept.

$$(0, 48)$$



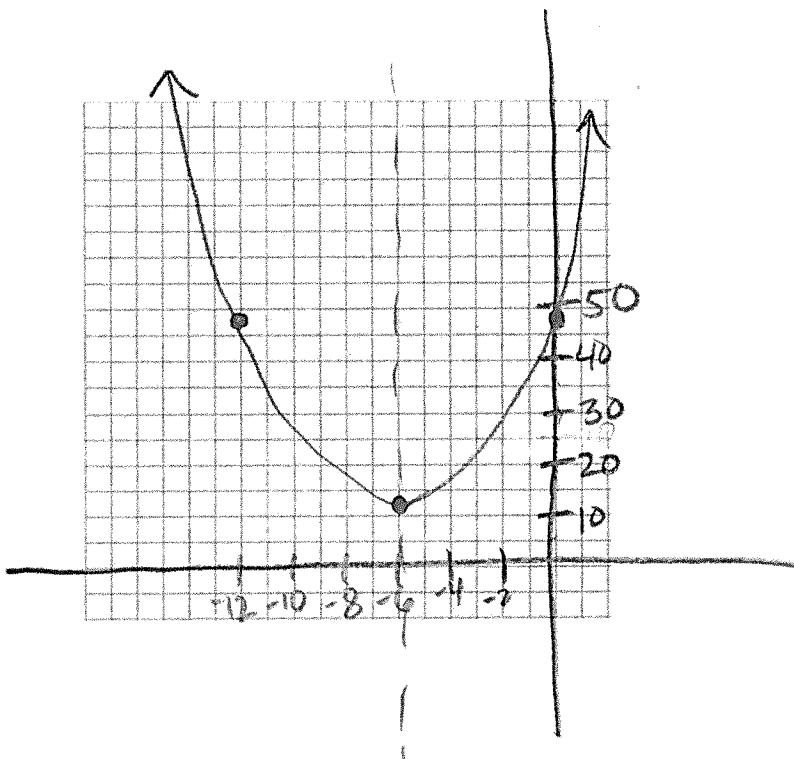
d) Find the x intercept(s).

~~$$0 = x^2 + 12x + 48$$~~

~~$$x = \frac{-12 \pm \sqrt{(12)^2 - 4(1)(48)}}{2} = \frac{-12 \pm \sqrt{144 - 192}}{2}$$~~

NO x intercepts

e) Draw a sketch of the graph.



11. A basketball is shot upward at 36 ft/s from a height of 6 feet. The height of the ball, h , at time t is given by the equation:

$$h(t) = -16t^2 + 36t + 6$$

Answer the following questions. Round all final answers to 2 decimal places (the nearest hundredth).

a) How high is the ball after 2 seconds?

$$h(2) = -16(2)^2 + 36(2) + 6 = -16(4) + 72 + 6 = -64 + 72 + 6 = 14 \text{ feet}$$

b) What is the maximum height the ball reaches?

$$x = \frac{-36}{2(-16)} = \frac{-36}{-32} = 1.125$$

$$h(1.125) = -16(1.125)^2 + 36(1.125) + 6 = -16(1.266) + 40.5 + 6 = -20.256 + 40.5 + 6 = 26.244$$

$$26.25 \text{ feet}$$

c) How long will it take to reach that maximum height?

$$1.125 \text{ seconds}$$

d) If the rim of the basketball hoop is 10 feet off the ground, how long will it take to hit the rim?

$$\begin{array}{r} 10 = -16t^2 + 36t + 6 \\ -10 \qquad \qquad \qquad -10 \\ \hline 0 = -16t^2 + 36t - 4 \end{array}$$

$$x = \frac{-36 \pm \sqrt{(36)^2 - 4(-16)(-4)}}{2(-16)} = \frac{-36 \pm \sqrt{1296 - 256}}{-32} =$$

$$= \frac{-36 \pm \sqrt{1040}}{-32} = \frac{-36 + 32.25}{-32} = 0.12$$

(continues on next page)

$$\frac{-36 - 32.25}{-32} = 2.13 \text{ seconds}$$

$$h(t) = -16t^2 + 36t + 6$$

(continued)

e) If the ball misses the rim altogether, how long will it take the basketball to hit the ground?

$$-16t^2 + 36t + 6 = 0$$

$$X = \frac{-36 \pm \sqrt{(36)^2 - 4(-16)(6)}}{2(-16)} = \frac{-36 \pm \sqrt{1296 + 384}}{-32} =$$

$$= \frac{-36 \pm \sqrt{1680}}{-32} = \frac{-36 \pm 41}{-32} \begin{cases} \rightarrow \frac{-36+41}{-32} = \frac{5}{-32} = -0.16 \text{ seconds} \\ \rightarrow \frac{-36-41}{-32} = \frac{-77}{-32} = \boxed{2.4 \text{ seconds}} \end{cases}$$

f) What was the initial height of the basketball at the moment the player released it?

$$\boxed{6 \text{ feet}}$$

g) Give the contextual (real-world) domain and range for this function.

$$\text{Domain (seconds)} \quad [0, 1.125]$$

$$\text{Range (feet)} \quad [0, 26.25]$$

12. Determine if the following tables represent a linear, exponential, quadratic function and explain how you know. If it is none of them then please explain.

x	y
1	-4
2	-2
3	3
4	11
5	22

d days	N(d) cells
1	45,000
3	30,000
5	20,000
7	13,333

t months	P(t) dollars
3	28,000
7	21,000
11	14,000
15	7,000

Quadratic:
Constant Second Differences

Exponential:
Constant Multiplier

Linear:
Constant Rate of change

13. For each function, determine whether it is linear, exponential, quadratic or neither. Then identify the initial value.

(0,7)

a) $y = 2x^2 - 5x + 7$

Quadratic

$y = ax^2 + bx + c$

(0,1000)

b) $y = 1000(\frac{1}{5})^x$

Exponential

$y = ab^x$

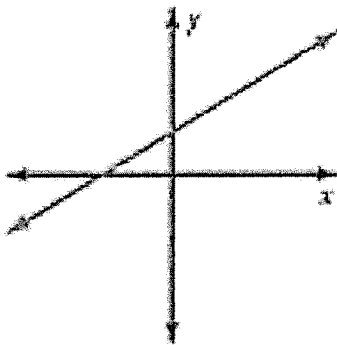
(0,22)

c) $y = -4x + 22$

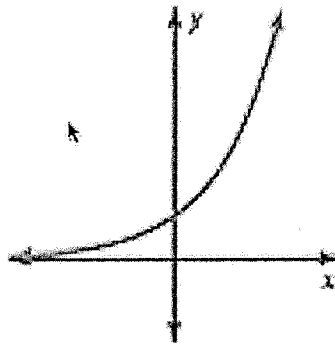
Linear

$y = mx + b$

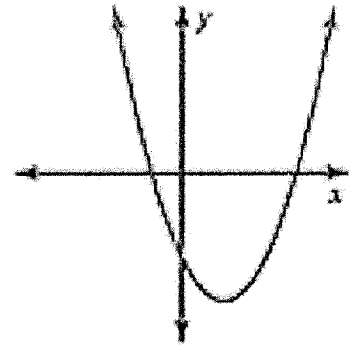
14. Identify if each is the graph of a linear, exponential or quadratic function. Then write an equation that could represent each graph.



Linear
positive slope
positive initial value



Exponential
 $a > 0$ (initial value)
 $b > 1$ (growth)



Quadratic
 $a > 1$
 $c < 0$ (initial value)